Interior-point methods for mathematical optimization

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INTRODUCTION

- The area of IPM has been one of the liveliest in mathematical programming in the last two decades. These techniques were primarily in the form of barrier methods, widely used during the 1960s for problems with nonlinear constraints, their use for the fundamental problem of linear programming was unthinkable because of the total dominance of the simplex method. During the 1970s, barrier methods were superseded, nearly to the point of oblivion, by newly emerging and seemingly more efficient alternatives such as augmented Lagrangian and sequential quadratic programming methods. By the early 1980s, barrier methods were almost universally regarded as a closed chapter in the history of optimization
- In 1984 Narendra Karmarkar announced a fast polynomial-time interior method for linear programming; in 1985, a formal connection was established between his method and classical barrier methods. Since then, interior methods have continued to transform both the theory and practice of constrained optimization.

Semi-definite and conic programming

- Semidefinite programming may be viewed as a generalization of linear programming, where the variables are $n \times n$ symmetric matricies, denoted by X, rather than n-vectors. In SDP we wish to minimize an affine function of symmetric matrix X subject to linear constraints and semidefinite constraints, the latter requiring that "X must be positive semidefinite". This typically written as $X \succeq 0$ that resembles inequality constraints in continuous optimization.
- Many extra complications arise in SDP. For example, the feasible region defined by constraints is not polyhedral, so there is no analogue of the simplex method.
- Nesterov and Nemirovski showed that function $\log \det x$ is self-concordant for SDP, which means that SDP can be solved in polynomial time via sequence of barrier subproblems parametrized by μ
- Conic programming is a subclass of convex optimization that deals with optimization problems where the feasible region is defined by a convex cone. It generalizes linear programming and encompasses several important types of problems, including quadratic programming and semidefinite programming.

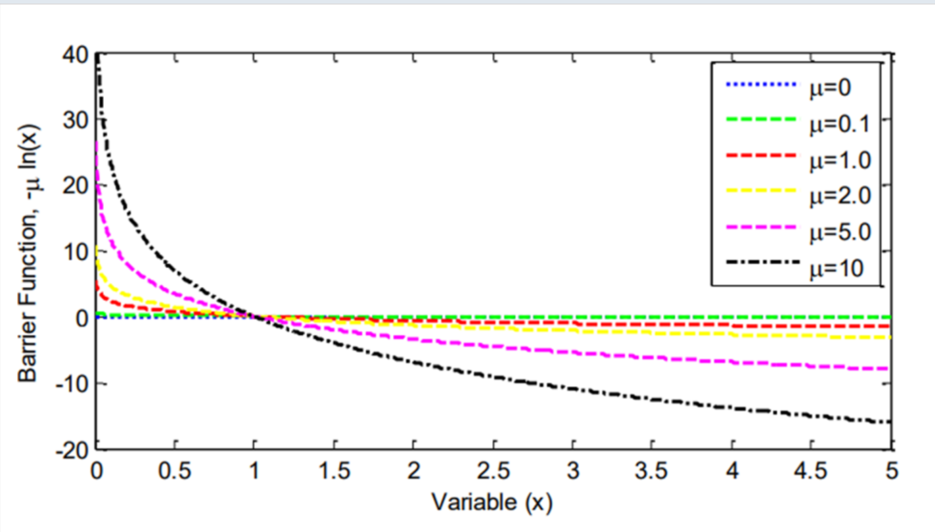
Early development and key Inspirations

- Karmarkar's method main features:
- Faster than any other for large scale programs
- Polynomial time convergence
- Ideas can be utilized in development of polynomial time algorithms for other optimization problems

Main ideas behind:

1. Iterative process starts from centre of feasible region to steepest decent direction
2. Used transformations in order to place current point near the center

- Include logarithmic barrier term in objective function f:
- $B(x, \mu) = f(x) - \mu \sum_{j=1}^{m} \ln c_j(x)$,
- Here μ is a parameter, as converges to zero to the minimum of $B(x, \mu)$ should converge to a solution of COP.



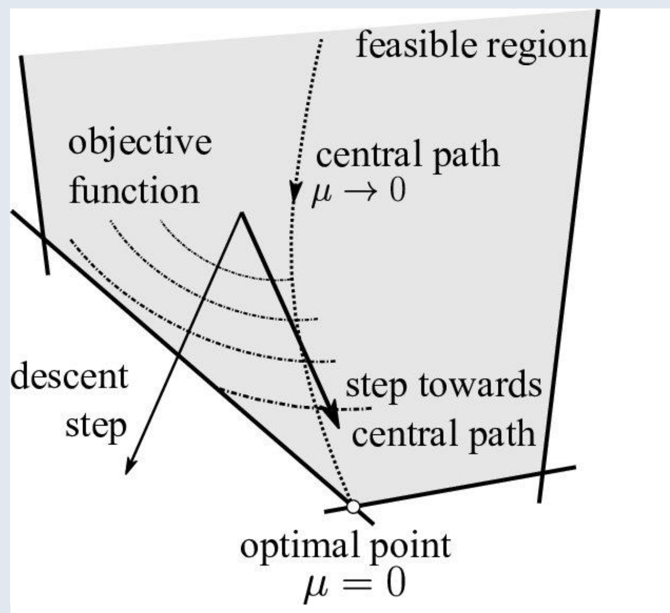
Concluding remarks and feature research

- Semidefinite and conic programming is an extremely lively research area today, producing new theory, algorithms, and implementations.
- IPM`s is a powerful tool in optimization, allowing for the efficient handling of a wide variety of real-world problems with structured constraints and objectives. Their adaptability and robustness make them particularly appealing for tackling complex real-world problems
- Numerous individual papers exist in the fields of semidefinite programming and conic programming, require thorough examination. Currently, there is no unified framework for applying interior point algorithms to various optimization problems with constraints. As a result, our team intends to integrate different types of interior point methods to evaluate their effectiveness on low-dimensional problems and extend these approaches primarily to large-scale issues.

Primal-Dual

- Iterative estimation of objective function proposed by Todd and Burrel solving dual for LP problem:
  $\max\{b^T y : A^T y + s = c, s \geq 0\}$,
  And also for standard LP from
- $\min\{\vec{c}^T * \vec{x}\}$ *such that* $Ax = 0,\ e^T x = 1, x \geq 0$,
- The main goal is to replace complementarity condition with parametrized condition $xs = \mu e, \mu >= 0$
- Advantages:
- More efficient than barrier in cases of high accuracy is needed
- often exhibit superliner asymptotic convergence
- search directions can be interpreted as Newton directions for modified KKT conditions
- could start at infeasible point
- cost per iteration same as barrier method
- Short and long step methods

Central path a new class of IPMs. These methods don't use Newtos direction, instead they use steepest decent direction for a so-called self-regular barrier function



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