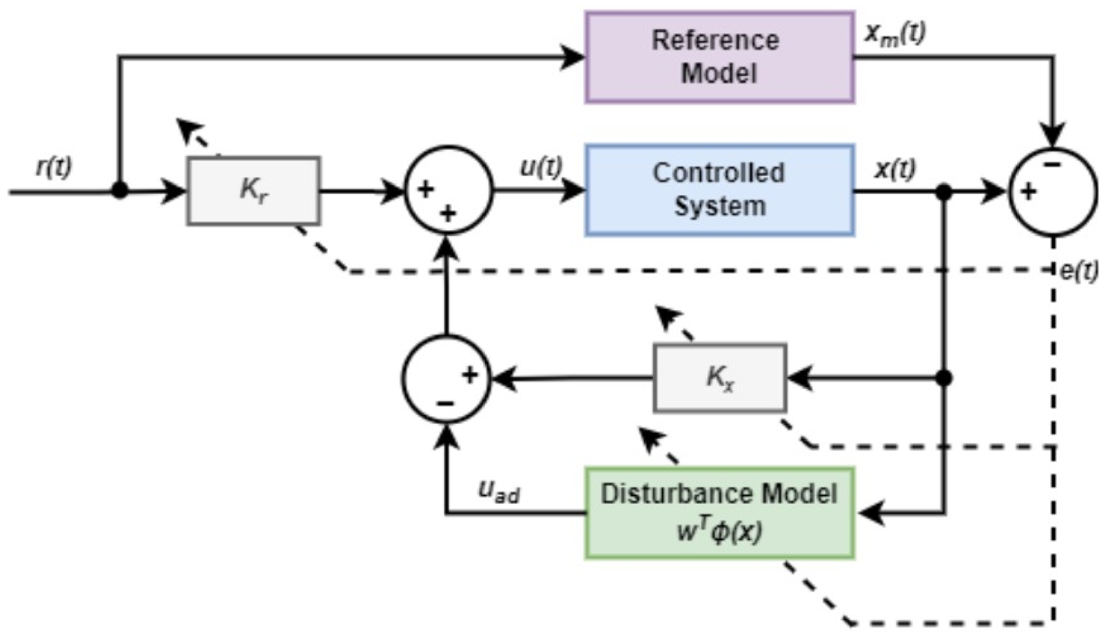
Longitudinal Movement Adaptive Control

Annotaion

Model Reference Adaptive Control (MRAC) computes control actions to enable an uncertain controlled system to follow the behavior of a reference model. This work explores the application of this method for longitudinal speed tracking in autonomous vehicles. The core idea of the method involves adding an adaptive component to the control strategy, with parameters that are adjusted through feedback. Various methods for constructing the adaptive part of the control are proposed, including the use of specialized tables. This approach aims to enhance the performance and robustness of vehicle speed control in uncertain conditions



MRAC Theory

uncertainty

assumption

$$\dot{x}(t) = Ax(t) + B\Lambda\left(u + \delta(x(t))\right)$$

$$\begin{cases} \delta(x) = w^T\theta_0(x) \\ w \in R^{s\times m} \text{ - unknown constant weights matrix} \\ \theta_0(x) \text{ - known basis function} \end{cases}$$

- $x(t) \in R^n$ - state vector
- $u(t) \in R^m$ - control command
- $A \in R^{n\times n}$ - known systems matrix
- $B \in R^{n\times m}$ - known input matrix
- $0 < \Lambda \in R^1$ - control effort uncertainty gain
- $\delta(x(t)) : R^n \to R^m$

Goal is to find such control law $u = u(.)$
that controllable output $y = Cx \quad C \in R^{1\times n}$
follows along some reference signal $r = r(t) \in R^1$

$$e = x - r, e \to 0$$

Nominal System

$$\dot{x}(t) = Ax(t) + Bu$$

Nominal Control

$$u_n = -Kx + K_{ff}r$$

where $K \in R^{N\times 1}$ is the feedback control matrix and
$K_{ff} \in R^1$ is the feedforward gain.

Control law

$$u = u_n + u_a$$
$$u_a = \hat{w}^\top\theta(x) = \hat{w}^\top\begin{pmatrix}\theta_0(x) \\ u_n\end{pmatrix}$$
$$\dot{\hat{w}} = -\gamma\theta(x)B^\top Pe$$

$V(x)$ - Lyapunov function

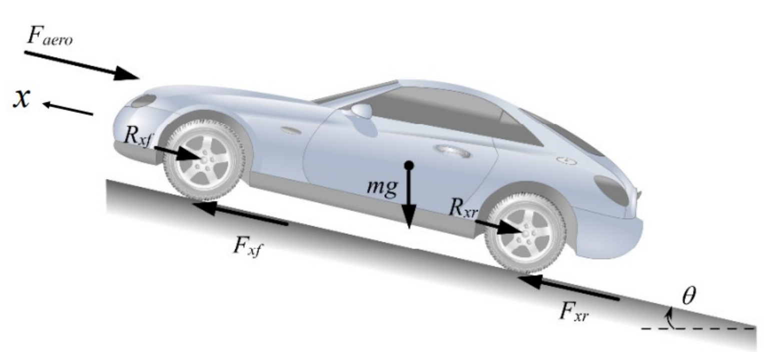
$$V(x) = e^\top Pe + tr\left[\left(\hat{w}\Lambda^{\frac{1}{2}}\right)^\top\left(\hat{w}\Lambda^{\frac{1}{2}}\right)\right]$$
$$\dot{V} = -e^\top Re \le 0$$

Forum on Robotics & Control Engineering (FoRCE, http://force.eng.usf.edu/) Seminar Series: "Model Reference Adaptive Control Fundamentals" (Dr. Tansel Yucelen)

Car longitudinal motion model

$$m\dot{v} = F_x - F_{aero} - R_x - mg\sin\phi$$
$$J\dot{\omega} = T_{acc}(u > 0) + T_{dec}(u < 0) - F_x r_{eff}$$

- $F_x = C_\sigma\sigma$ - longitudinal tire force
- $\sigma = \frac{r_{eff}w}{v} - 1$ - slip ratio
- $\dot{T}_{acc} = f_{acc}(u),\ u > 0$ - torque transmitted from acceleration system
- $\dot{T}_{dec} = f_{dec}(u),\ u < 0$ - torque transmitted from braking system



$\dot{v} = u - \hat{F}_{aero} - \hat{R}_x - g\sin\phi$ - Nominal system

$$u = \underbrace{g\sin\phi + \dot{v}_{ref}}_{\text{FeedForward}} + \underbrace{\hat{F}_{aero} + \hat{R}_x + k_p(v - v_{ref})}_{\text{FeedBack}} \rightarrow \dot{e} = -k_p e \rightarrow e \to 0$$

Control law

$$\begin{cases} u = \underbrace{g\sin\phi + acc_{ref} + k_p(v - v_{ref})}_{u_n} + \underbrace{\hat{w}^T\theta(v, u_n)}_{u_a} \\ \dot{\hat{w}} = -\gamma\theta(v, u_n)e \end{cases}$$

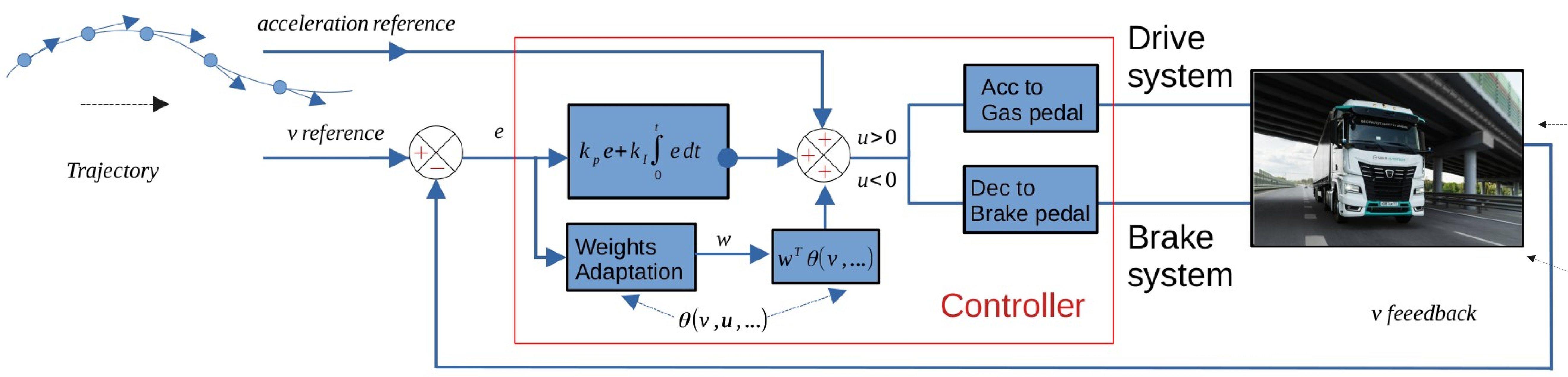
Uncertainty compensation

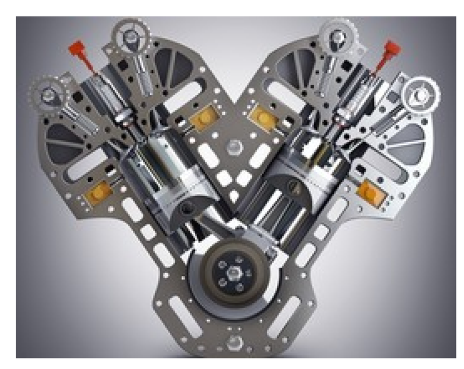
$$\theta(v, u_n) = \begin{pmatrix} 1 \\ u_n[u_n > 0] \\ u_n[u_n < 0] \\ v/v_{max} \\ 1 - 2(v/v_{max})^2 \ldots \end{pmatrix}^T \qquad u_a = \begin{pmatrix} w^1 \\ w^2 \\ w^3 \\ \ldots \end{pmatrix}^T \begin{pmatrix} 1 \\ u_n[u_n > 0] \\ u_n[u_n < 0] \\ \ldots \end{pmatrix}$$

Acceleration error compensation

Deceleration error compensation

Chebyshev polinoms





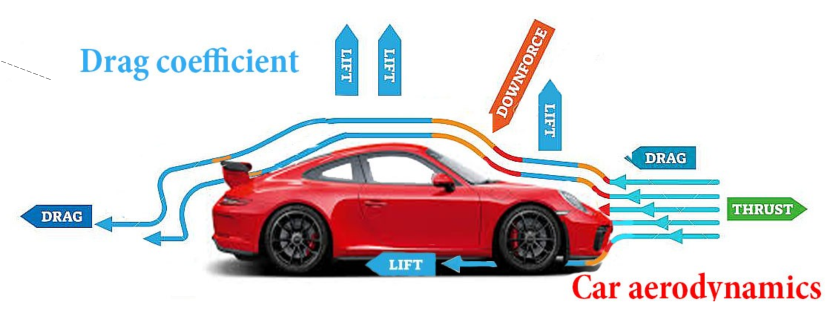
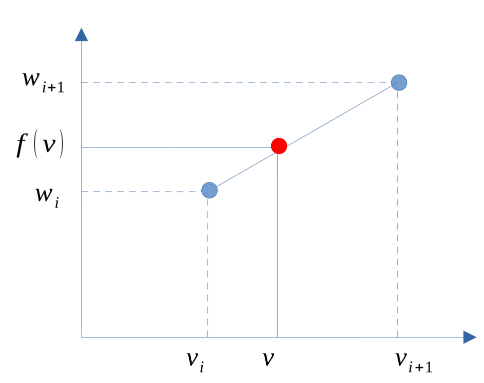


Table mrac

One dimentional table

$$f(V, v) = \begin{bmatrix} W \in R^m, V \in R^m \\ 0 \le i < m \\ \forall i : v^i \in V, v^i < v < v^{i+1} \end{bmatrix} = \begin{pmatrix} w^{i+1} \\ w^i \end{pmatrix}^T \underbrace{\begin{pmatrix} v^{i+1} - v \\ v - v^i \end{pmatrix} \frac{1}{(v^{i+1} - v^i)}}_{\theta(v, u_n)}$$

$$u_a = f(W^1, V, v) + f(W^2, V, v)u_n$$



One dimentional table

$$f(W, V, v) = \begin{bmatrix} W \in R^{m\times k}, \\ V \in R^k, U \in R^m \\ 0 \le i < m, 0 \le j < k \\ \forall i, j : v^i \in V, u_n^j \in V, \\ v^i < v < v^{i+1}, \\ u^i < u_n < u^{i+1} \end{bmatrix} = \begin{pmatrix} w_i^j \\ w_{i+1}^j \\ w_i^{j+1} \\ w_{i+1}^{j+1} \end{pmatrix}^T \underbrace{\begin{pmatrix} (v^{i+1} - v)(u^{j+1} - u_n) \\ (v - v^i)(u^{j+1} - u_n) \\ (v^{i+1} - v)(u_n - u^j) \\ (v - v^i)(u_n - u^j) \end{pmatrix} \frac{1}{((v^{i+1} - v^i)(u^{j+1} - u^j))}}_{\theta(v, u_n)}$$

$$u_a = f([W^1, V, U], v, u_n) + f([W^1, V, U], v, u_n)u_n$$

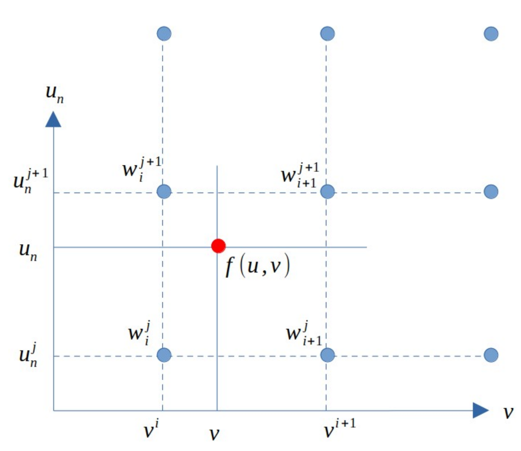


Table adaptive control relies on linear or bilinear interpolation from tables, with coefficients that are adjusted adaptively according to MRAC control law. The following graphs illustrate the simulation of vehicle speed tracking, taking into account the effects of tire characteristics and the nonlinear behavior of the engine. The method is based on the assumption that the system's characteristics vary at different speeds and with different levels of throttle and brake pedal engagement.