Dynamic system reconstruction from multivariate time series via multilinear map and time delay embedding

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Denis Tikhonov [1] Vadim Strijov [1]

[1]Forecsys LLC, Moscow, Russia

Introduction

Embedding methods are commonly used to analyse time series whose full system state cannot be fully or directly observed. One common class of embedding methods — time delay embedding — requires the careful selection of embedding lags.

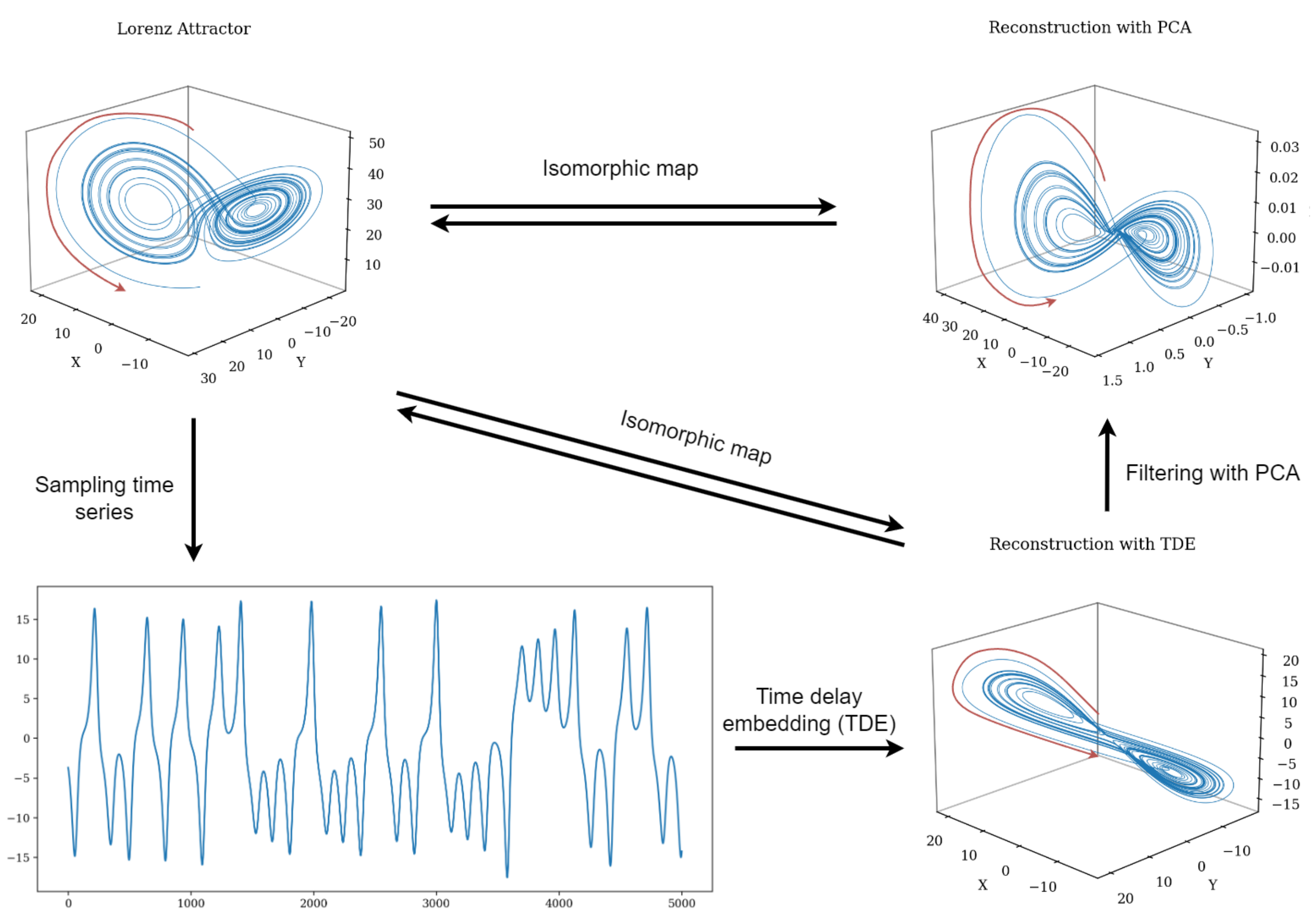

Figure 1. Scheme of attractor reconstruction with time delay embedding (TDSE) and principal component analysis (PCA).

The dimension of the space is the length of a vector with previous values in time.

- As the dimension of the phase space increases, the distances between the points of the trajectory **tend to the constant value**.
- The proper dimension is **significantly less** than the dimension of the original phase space.
- That makes **distances uninformative and unstable** due to the curse of dimensionality.

Also, it assumes that a more stable and robust model is possible in the subspace than in the original one.

Key idea

To prevents the curse of dimensionality various method is used. The most common method for such analysis is the principal component method (PCA). This is a linear method. To extend it, it is proposed to use the tensor method for characterizing the state of the multidimensional data.

The key idea is to expand dynamic system model in addition to the time delay embedding to multivariate time serise. Such model also **prevents the loss of higher-order information**.

Proposed method

Let s_x, s_y, s_z be the time series of acceleration along each of three axes. A signal from each axis separately restores the attractor of the dynamic system according to Taken's theorem using time delay embedding. Also, there are linear maps between each variable using rotation and stretching (i.e. affine transformations)

$$S_x = I^\mathsf{T} S_x, \quad S_x = W_y^\mathsf{T} S_y, \quad S_x = W_z^\mathsf{T} S_z, \qquad (1)$$

where S_x, S_y, S_z are trajectory matrices in initial phase space, $W_y^\mathsf{T}, W_z^\mathsf{T}$ are the transformation matrices, I is an identity matrix. Thus, the multilinear model is modified as follows:

$$x_t = A \times_1 (I^\mathsf{T} s_{x\,t}) \times_2 (W_y^\mathsf{T} s_{y\,t}) \times_3 (W_z^\mathsf{T} s_{z\,t}) = \hat{A} \times_1 s_{x\,t} \times_2 s_{y\,t} \times_3 s_{z\,t}, \qquad (2)$$

where $\hat{A} = A \times_1 I^\mathsf{T} \times_2 W_y^\mathsf{T} \times_3 W_z^\mathsf{T}$is modified dynamic tensor, $s_{x\,t}, s_{y\,t}, s_{z\,t}$ are state variable vectors from each axis at time t.

The tensor $\hat{A}$ allows to select not only the main components, as in case of PCA for univariate time series, but filter them according to multilinear dependencies with other time series.

Experiment

This dataset includes time-series data generated by accelerometer and gyroscope sensors. It is collected with an iPhone 6s kept in the participant's front pocket using SensingKit. All data collected in 50Hz sample rate. A total of 24 participants in a various of gender, age, weight, and height performed six activities in the same environment and conditions: downstairs, upstairs, walking, jogging, sitting, and standing.

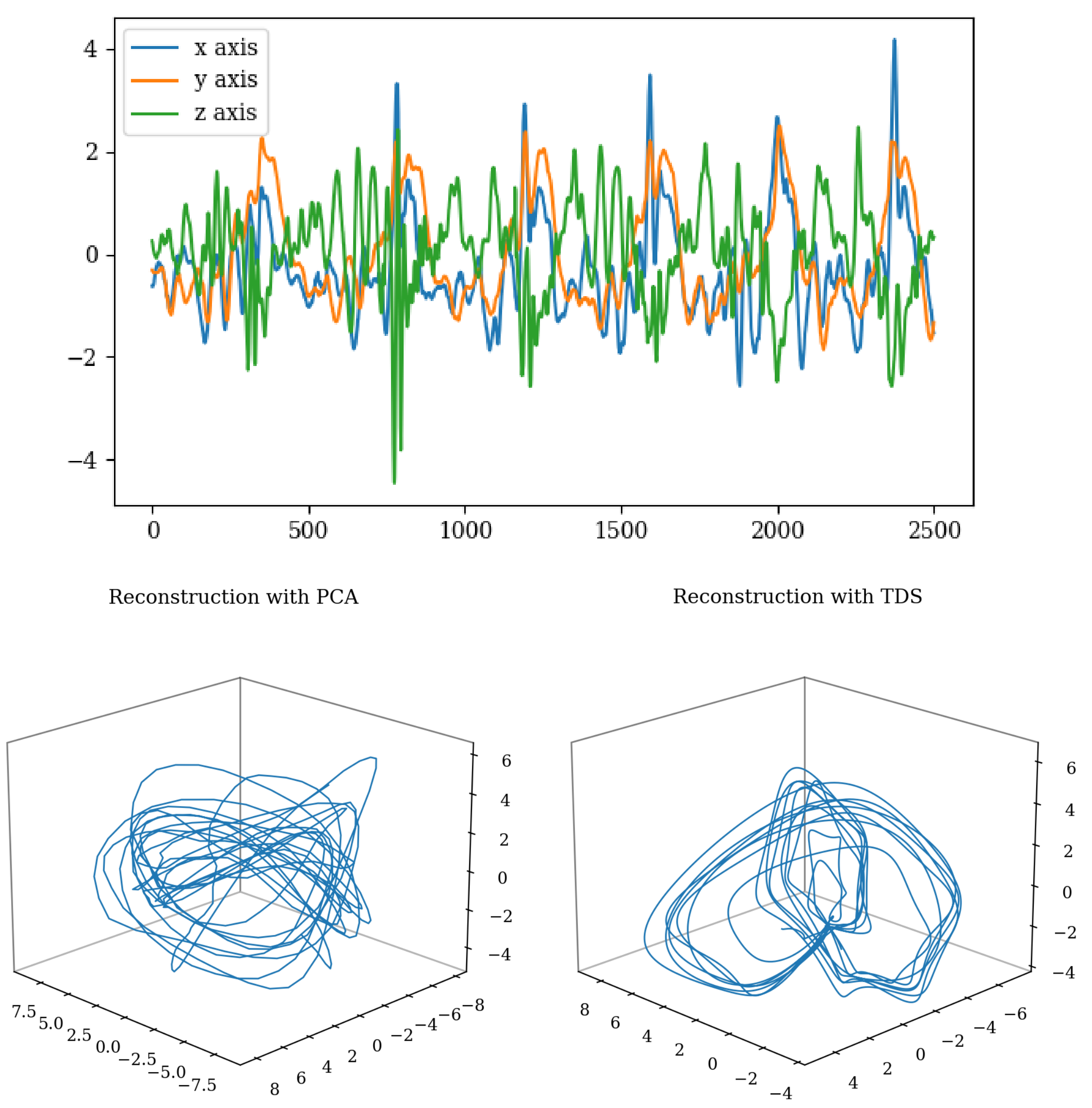

Figure 2. Time series sample, reconstruct phase space with PCA and TDS of activity upstairs

Thus, on several real time series it was shown that in the case of a linear dependence, the proposed method allows to obtain more interpretable results and reduces the number of intersections. In the case of clearly nonlinear dependences, the result becomes complex.

Conclusion/Future perspectives

This paper solves the problem of dimensionality reduction for the phase reconstruction of multivariate time series. The result of the work is a generalization tensor dynamical system in the case of multivariate time Proposed method retains the required properties and reproduces the type of the original attractor with a high accuracy in linear case.

There are three main directions for future work:

- to take into account **nonlinear relationships** through, for example, autoencoders and nonlinear activation functions;
- to increase computational efficiency with a more complex approach which will use not all available **components**, but those with **the highest correlation in the multivariate time series**;
- to optimize **the construction of the tensor representation** due to the exponential growth of the number of parameters in the case of a larger number of time series.

References

[1] Can Chen.
On the stability of multilinear dynamical systems, 2022.

[2] Can Chen, Amit Surana, Anthony Bloch, and Indika Rajapakse.
Multilinear Time Invariant System Theory, page 118–125.
Society for Industrial and Applied Mathematics, January 2019.

[3] N. H. Packard, J. P. Crutchfield, J. D. Farmer, and R. S. Shaw.
Geometry from a time series.
Phys. Rev. Lett., 45:712–716, Sep 1980.