Navigating Byzantine Attacks in Distributed Learning: Trial Function Methodology

ICOMP 2024

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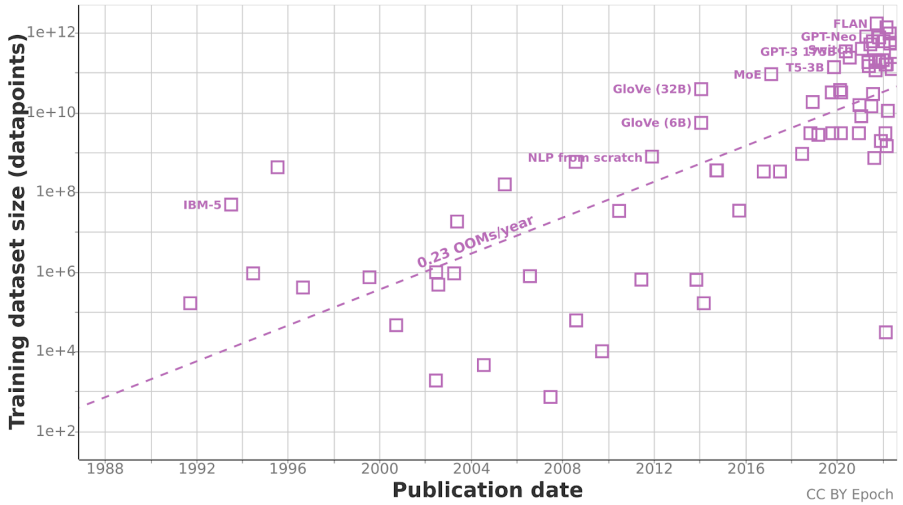
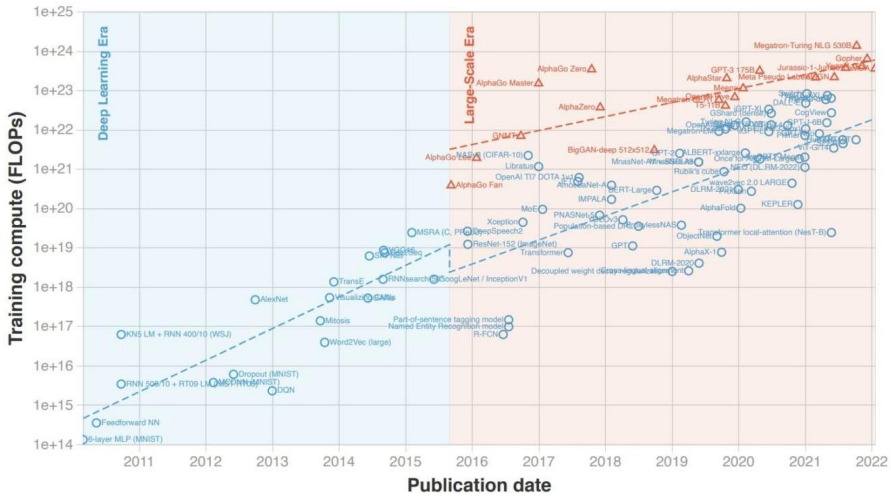
October 10, 2024

Modern trends in machine learning

- Exponential growth in model sizes and data volumes.





Varieties of distributed learning

- Cluster learning (big players): training within one large and powerful computing cluster
- Collaborative learning (all players): pooling computing resources over the Internet

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- Cluster learning (big players): training within one large and powerful computing cluster
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- Formally (horizontal, offline distributed learning):

$$\min_{x \in \mathbb{R}^d} \left[f(x) := \frac{1}{M} \sum_{m=1}^{M} f_m(x) := \frac{1}{M} \sum_{m=1}^{M} \frac{1}{n_m} \sum_{i=1}^{n_m} l(g(x, a_i^m), b_i^m) \right],$$

  where x – model weights, g – model, l – loss function.
- The data is shared among M computational devices, each device m has its own local subsample $\{a_i^m, b_i^m\}_{i=1}^{n_m}$ of size n_m.

Communicate centralized via server

Let us look at an example of how classical SGD becomes distributed.

Алгоритм 1 Centralized Distributed GD

Вход: Stepsize $\gamma > 0$, starting point $x_0 \in \mathbb{R}^d$, number of iterations K

for $k = 0, 1, \ldots, K-1$ **do**
  Send x_k to all workers ▷ server
  for $m = 1, \ldots, M$ in parallel **do**
    Receive x_k from server ▷ workers
    Compute gradient $g_{m,k} = \nabla f_m(x_k, \xi_{m,k})$ ▷ workers
    Send $g_{m,k}$ to server ▷ workers
  end for
  Receive $g_{m,k}$ from all workers ▷ server
  Compute $g_k = \frac{1}{M} \sum_{m=1}^{M} g_{m,k}$ ▷ server
  $x_{k+1} = x_k - \gamma g_k$ ▷ server
end for

Problem

- We can't trust all the workers. They can be lazy or even adversarial.

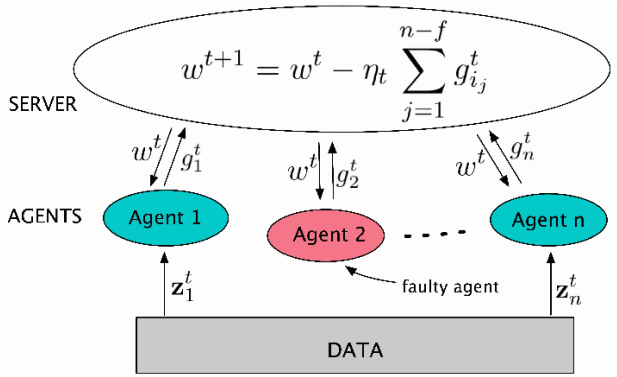
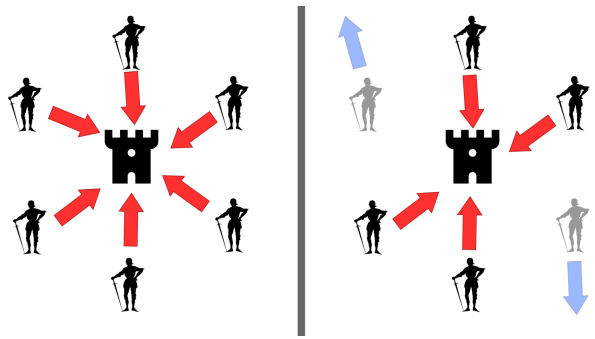


Figure: Distributed computing with adversarial worker

- Classical problem of Byzantine generals – find applications even in modern aviation.



Related work

- Classical methods like SGD, ADAM, and SCAFFOLD fail with even one Byzantine worker.

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- Classical methods like SGD, ADAM, and SCAFFOLD fail with even one Byzantine worker. **Question:** why?
- Mentioned Distributed SGD:

$$g = \frac{1}{M}\sum_{m=1}^{M} g_m.$$

If Byzantine worker j send "gradient" $g_j = -\sum_{m\in[M]/j} g_m$, learning process stops.

Related Work

Coordinate-wise median:

$$[\mathrm{CM}(g_1, \ldots, g_M)]_j = \mathrm{median}([g_1]_j, \ldots, [g_n]_j).$$

Intuition: the median value of the gradients for each coordinate.

Krum: Suppose subset $S \subset [M]$ with size at least $(M - \delta M - 2)$, where among M workers, at least $(1 - \delta)n$ is the honest ones.

$$\mathrm{Krum}(g_1, \ldots, g_M) = \arg \min_{g_i} \min_{S} \sum_{j \in S} \|g_i - g_j\|_2^2.$$

Intuition: the closest to the average gradient after excluding $\delta n + 2$ furthest away gradients.

Trimmed Mean: For each coordinate j compute sotring sort_j.

$$[\mathrm{TM}(g_1, \ldots, g_M)]_j = \frac{1}{M - 2\delta M} \sum_{i=\delta M}^{M - \delta M} [g_{\mathrm{sort}_j(i)}]_j.$$

Intuition: the average after excluding δM smallest and highest gradient values for each coordinate.

Related Work

Client Momentum with Central Clipping Aggregation:

$$\mu_{m,k} = (1-\beta)g_{m,k} + \beta\mu_{m,k-1}$$

$$c_{m,k} = (\mu_{m,k} - \hat{\mu}_{k-1}) \cdot \min\left(1, \tau/\|\mu_{m,k} - \hat{\mu}_{k-1}\|\right)$$

$$\hat{\mu}_k = \hat{\mu}_{k-1} + 1/M \sum_{m=1}^{M} \mu_{m,k}$$

$$x_{k+1} = x_k - \gamma\hat{\mu}_k$$

Intuition: clip/cut large deviations from the past average.

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RECCESS:

$$S(g_m, g_j) = \frac{\langle g_m, g_j \rangle}{\|g_j\| \ cdot \|g_m\|}$$

$$w_{m,k} = \frac{\sum_{j=1}^{M} S(g_m, g_j)}{\sum_{j=1}^{M}\sum_{i=1}^{M} S(g_i, g_j)}$$

$$x_{k+1} = x_k - \gamma \sum_{m=1}^{M} w_{m,k} g_{m,k}$$

Intuition: to see if all possible gradient pairs are same orientated.

Setup

In the distributed setting, we are guaranteed data homogeneity, which means that each distribution $\mathcal{D}_i$ coincides with the distribution of the entire training sample $\mathcal{D}$, where $\mathcal{D}_i$ is an unknown distribution of the training sample data on the i-th device.

Stochastic minimization problem

This allows us to move to the classical formulation of the problem:

$$\min_{x \in \mathbb{R}^d} \left[f(x) = \mathbb{E}_{\xi \sim \mathcal{D}} \left[f_\xi(x) \right] \right]. \tag{1}$$

GofM is the number of good workers.

Setup

To tackle Byzantine attacks, we introduce a trial loss function $\hat{f}$. In setting (1), we sample a subset from $\mathcal{D}$ to calculate $\hat{f}$. The distribution of the trial function matches $\mathcal{D}$, making it an honest estimator of f. The larger the sample, the better $\hat{f}$ approximates f.

Formal definition

Formally, the trial function is as follows:

$$\hat{f}(x) = \frac{1}{N} \sum_{i=1}^{N} f_i(x),$$

where N is the number of samples in $\hat{f}$.

Assumptions

Assumption 1

The function $\hat{f}$ is L-smooth, i.e., it satisfies $\|\nabla \hat{f}(x) - \nabla \hat{f}(y)\| \leq L\|x - y\|$ for any $x, y \in \mathbb{R}^d$.

Assumption 2

- Convexity: the function $\hat{f}$ is convex, i.e., it satisfies $\hat{f}(y) \leqslant \hat{f}(x) + \langle \nabla \hat{f}(x), y - x \rangle$ for any $x, y \in \mathbb{R}^d$.
- Non-convexity: the function $\hat{f}$ has a (may be not unique) minimum, i.e., $\hat{f}^* = \inf\limits_{x \in \mathbb{R}^d} \hat{f}(x) > -\infty$.

Assumptions

Assumption 3

Each worker $i \in G$ has access to an independent and unbiased stochastic gradient with (depending on the either homogeneity or heterogeneity of the data distribution): $\mathbb{E}[g_i(x, \xi_i)] = \nabla f(x)$ and its variance is bounded by σ^2_{hom} : $\mathbb{E}[\|g_i(x, \xi_i) - \nabla f(x)\|^2] \leq \sigma^2_{\text{hom}}$;

Assumption 4

Byzantine workers are assumed to be omniscient, i.e., they have access to the computations made by the rest of the workers.

First idea

- Indication of Trial Function Reduction (ITR-SGD): Weights (trust score) are determined by how much the gradient, relative to those from other devices, reduces the trial function:

$$\omega_{m,k} = (1-\beta)\omega_{m,k-1} + \beta \frac{[\hat{f}(x_k) - \hat{f}(x_k - \gamma g_{m,k-1})]_0}{\sum_{j=1}^{M}[\hat{f}(x_k) - \hat{f}(x_k - \gamma g_{m,k})]_0}$$

$$x_{k+1} = x_k - \gamma \sum_{i=1}^{n} 1_{[\hat{f}(x_k) - \hat{f}(x_k - \gamma g_{m,k}) > 0]} \cdot \omega_{m,k} \cdot g_{m,k}$$

ITR-SGD analysis

Theorem

For solving the problem described in (1)*, after* K *iterations of* ITR-SGD *with* $\gamma \leqslant \frac{1}{4L}$*, the following holds under Assumptions 1, 2.1, 3, 4*

$$\frac{1}{K}\sum_{t=0}^{K-1}\mathbb{E}\|\nabla f(x_k)\|^2 \leqslant \frac{2\mathbb{E}\left[\hat{f}(x_0)-\hat{f}(\hat{x}^*)\right]}{\gamma T}\cdot\frac{M}{\beta|G|}+4L\gamma\sigma_{hom}^2+\frac{1}{L\gamma}\zeta(N).$$

- First term – convergence of GD, but slow down by $fracM\beta|G|$.
- Second term – convergence of SGD with $|G|$ clients.
- Third term – $\hat{f} \approx f$ error.

Second idea

- Trial Function Minimization (TFM-SGD):
  Select optimal weights (trust scores) of the gradients on which the trial function is minimal:

$$\omega_k \approx \arg\min_{\omega \in \Delta} \hat{f}\left(x_k - \gamma \sum_{m=1}^{M} \omega_m g_{m,k}\right)$$

$$x_{k+1} = x_k - \gamma \sum_{m=1}^{M} \omega_{m,k} g_{m,k}$$

TFM-SGD analysis

Theorem

Suppose Assumptions 1, 2.2, 3, 4 hold. Then after T iteration of TFM-SGD *with $\gamma \leqslant \frac{1}{4L}$, it implies that*

$$\frac{1}{K}\sum_{k=0}^{K-1}\mathbb{E}\left\|\nabla f(x_k)\right\|_2^2 \leqslant \frac{2\mathbb{E}\left[\hat{f}(x_0)-\hat{f}^*\right]}{\gamma K} + \frac{4L\gamma}{|G|}\sigma_{hom}^2 + \frac{\zeta(N)}{\gamma L} + \frac{2\delta}{\gamma}.$$

- First term – convergence of GD, but slow down by $fracM\beta|G|$.
- Second term – convergence of SGD with $|G|$ clients.
- Third term – $\hat{f} \approx f$ error.
- Fourth term – subproblem error.

Third idea

- FineTuned-SGD:
  Look not at the loss function $\hat{f}$, but at the output of the model g on the trial data $\hat{D}$.

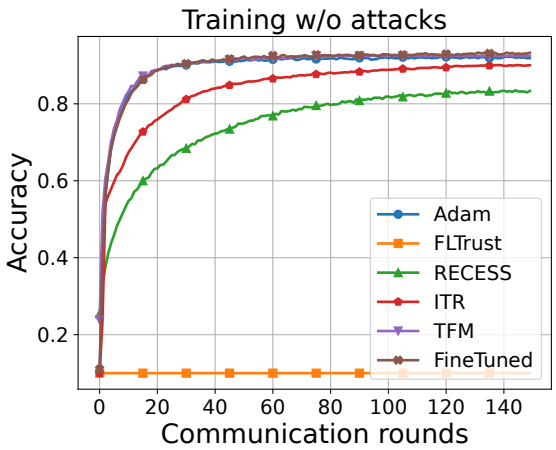
$$\omega_{m,k} = (1-\beta)\omega_{m,k-1} + \beta \frac{sim(m(x_k - \gamma g_{m,k}, \hat{D}), m(x_k - \gamma g_{0,k}, \hat{D}))}{\sum_{j=1}^{n} sim(m(x_k - \gamma g_{m,k}, \hat{D}), m(x_k - \gamma g_{0,k}, \hat{D}))}$$

$$x_{k+1} = x_k - \gamma \sum_{m=1}^{M} \omega_{m,k} g_{m,k}$$

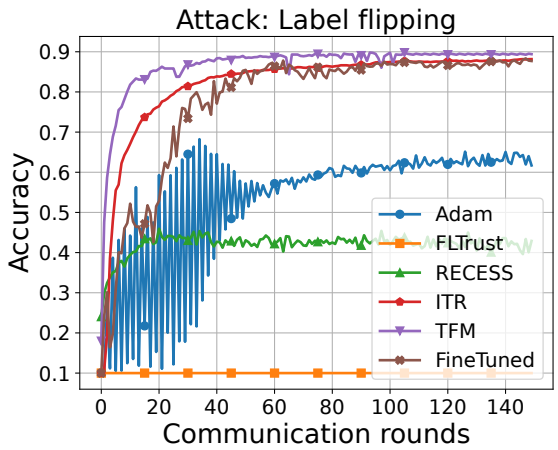
Experiments

- We conduct experiments on public CIFAR-10 image dataset.
- We use ResNet-18 model.

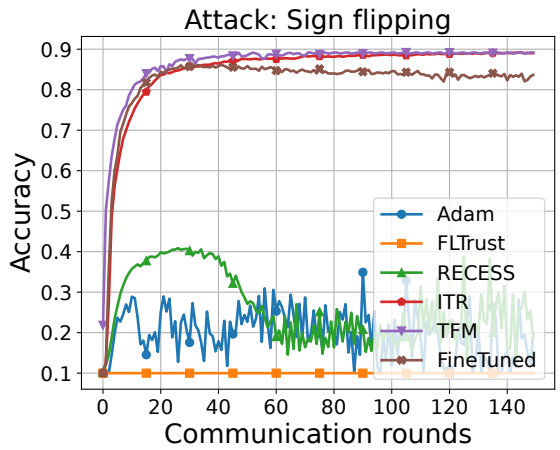
Experiments



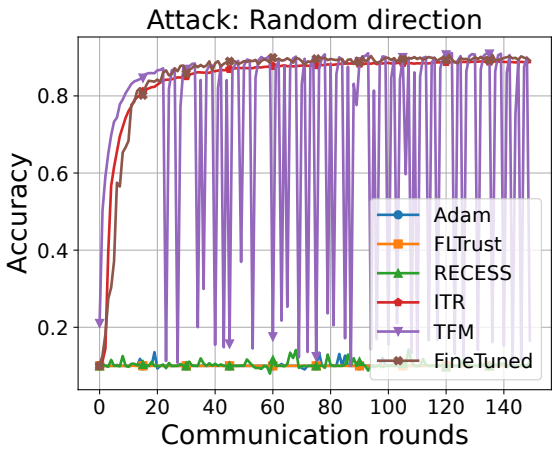
(a) Without Attack



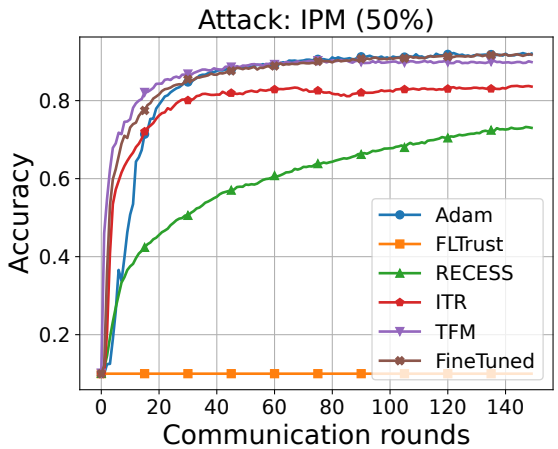
(b) Label Flip



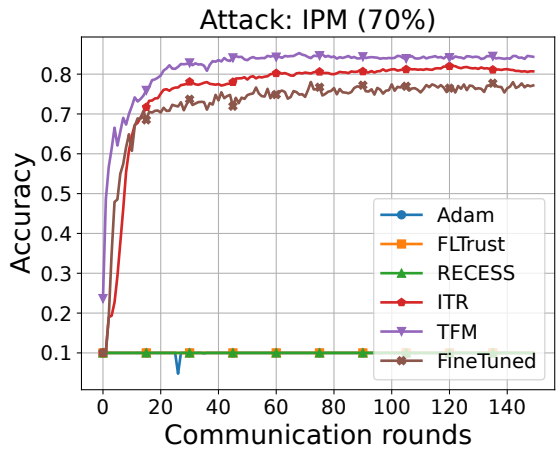
(c) Sign Flip



(d) Random Clients

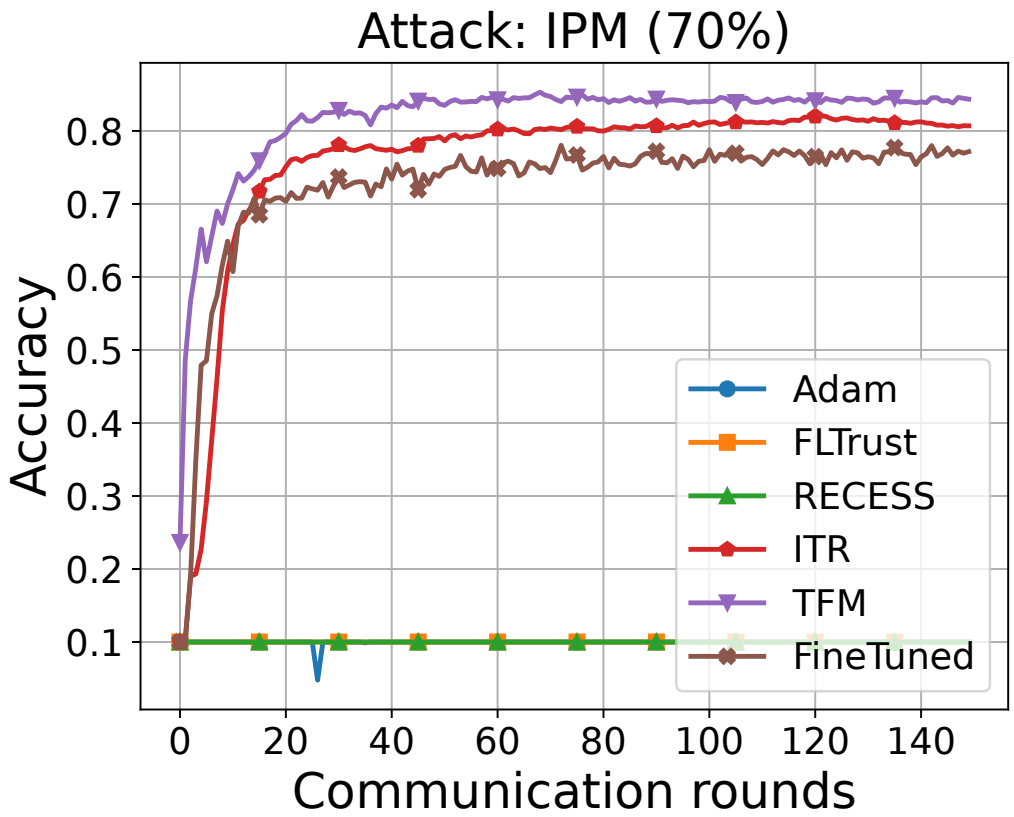


(e) IPM (50 %)



(f) IPM (70 %)

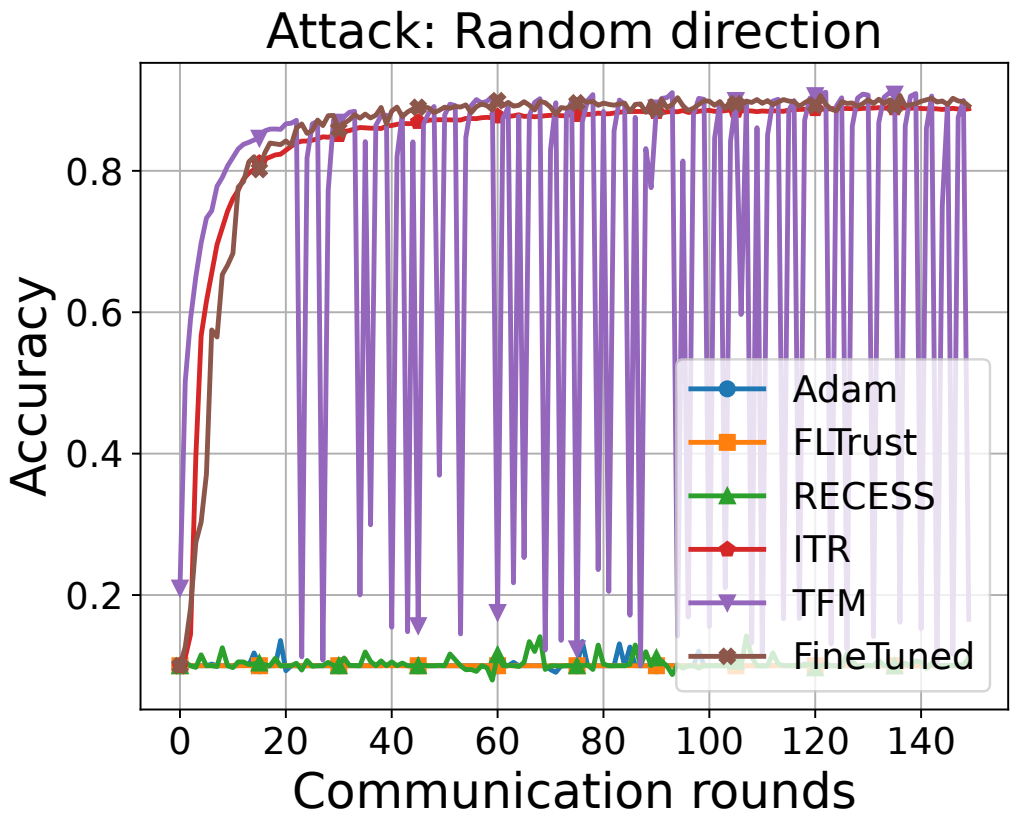
Experiments



(f) IPM (70 %)

Figure: ResNet18 on CIFAR-10 experiments results. Test accuracy in case of various attacks and Byzantine-tolerance techniques.

Experiments



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