Gravity Compensator for Prismatic Joints

Albert Demian
Center for Technologies in
Robotics and Mechatronics Components
Innopolis University
Innopolis, Russia
a.demian@innopolis.university

Alexandr Klimchik
Center for Technologies in
Robotics and Mechatronics Components
Innopolis University
Innopolis, Russia
a.klimchik@innopolis.ru

***Abstract*—We propose a concept for a gravity compensator designed for prismatic joints. The compensator depends on the suspension of linear springs together with transmission mechanisms to achieve constant application of force along the sliding span of the joint. We introduce the use of self-locking worm gears to ensure isolation of spring forces. A 2-DoF system which consists of a revolute and a prismatic joints is investigated with the proposed compensator. We introduce the use of pin-slot mechanism to transform rotational motion of the revolute joint into linear wire displacement. We introduce a design methodology of the compensator and selection of parameters to achieve static balancing. The results of simulation show complete compensation of gravity force leading to zero actuator effort.**

***Index Terms*—Prismatic joints , Static Balancing, Gravity Compensation, Manipulator Design**

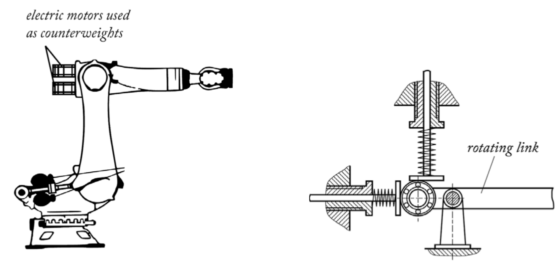
I. Introduction

Robots experience large static forces while operating in large works paces. These static forces are mainly generated by gravity which means a large part of the energy spent during operating goes to support robot's weight [1], [2].

Various approaches were proposed to compensate gravity. A classical approach is to use counter-weight as shown in Fig. 1a. This approach allows the manipulation of larger payloads, however it increases the total potential energy of the system [3]. Other approaches are based on auxiliary mechanisms with spring suspension similar to the one in Fig. 1b. The advantage of using springs is that they are light in weight hence, the increase of the system's potential energy is insignificant. Moreover, springs can store potential energy which reduces the energy required for operation. Many mechanisms for gravity compensation are summarized in [4]. More mechanisms are presented in detail in [5]–[14]. These mechanisms achieve different results between complete and partial compensation for links' weight.

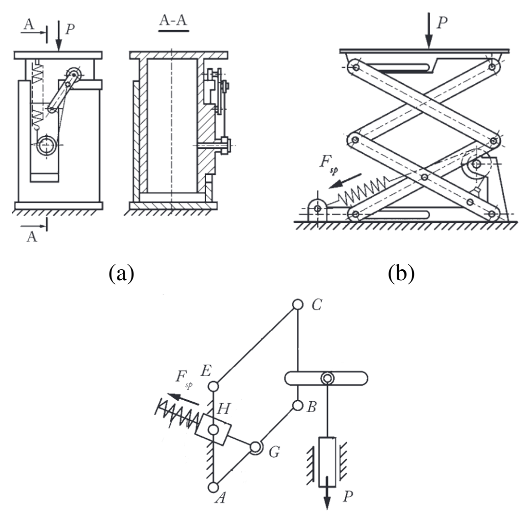
The majority of those mechanisms address gravity compensation for revolute joints. Moreover, a limited number of researches addressed compensation for prismatic joints. Gravity compensation for prismatic joints is addressed in [15]–[17]. The main drawback of these proposed mechanisms is their size if to be realized in design as shown in Fig. 2. A challenge regarding gravity compensation for prismatic joints is that the center of mass shifts accordingly with motion.

In this paper, we are proposing a preliminary concept of a gravity compensator for prismatic joints. The mechanism depends on spring tension together with a combination of pulleys and gear transmission. Moreover, this mechanism aims to compensate gravity force on prismatic joints at different orientations. Also, the proposed concept includes compensation of gravity for both joints in 2-DoF case where a prismatic joint is mounted on a revolute joint. The concept requires changing the design of manipulators to include the new components that we propose. The concept depends on analytical decoupling of



(a) Gravity compensation using counter-weight
(b) Gravity compensation using auxiliary mechanism

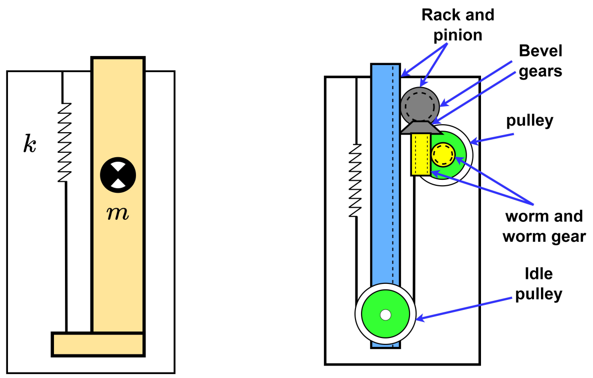
Fig. 1: Examples of gravity compensation



(a) (b)

(c)

Fig. 2: Examples of gravity compensators for prismatic joints [4]



(a) linear actuator at an arbitrary configuration with spring suspension

(b) A geometric representation of the constant-tension mechanism

Fig. 3: Gravity compensator for a prismatic joint in vertical configuration

effort terms and add equivalent spring-based components to produce counter-force. A combination of those components can analytically eliminate those decoupled terms.

II. Compensator for a 1-DoF prismatic joint at an arbitrary configuration

The goal of this section is to present a simple case of gravity compensation for a 1-DoF system of a prismatic joint. The aim here is to show how to compensate forces on a prismatic joint with a counter-force generated by a linear spring. Fig. 3a shows a geometric representation of a prismatic joint with the moving part of mass m at an arbitrary configuration. The mass induces a reaction effort τ in the actuator in the reverse direction to achieve equilibrium. Taking gravitational acceleration g pointing downwards, the actuator's effort can be as follows:

$$\tau = m\ g \tag{1}$$

where τ is the actuator's effort, m is the mass of the moving link and g is the gravitational acceleration.

To generate counter-force, we can add a spring with stiffness k connecting the moving part of the actuator to its base. Setting the spring with a proper pre-tension s_0 will generate a counter force $_{sp}$.

$$k\ s_0 = F_{sp} \tag{2}$$

We can achieve complete compensation by setting a proper pre-tension value s_0 to generate spring force F_{sp} that can counter-balance the link's weight.

$$s_0 = \frac{m\ g}{k} \tag{3}$$

As the prismatic joint performs linear motion, such compensator construction can not perform compensation at different joint configurations. This creates the need to design a mechanism that can keep this compensation force constant at any joint extension. A constant-tension mechanism is shown in Fig. 3b. A rack is coupled with the joint's slider and meshed with a pinion gear. The pinion gear is coupled with a bevel gear to transform the motion on perpendicular axes. A worm gear is coupled with the perpendicular bevel gear and meshed as input to a gear transmission to achieve motion locking. A pulley is coupled with the output of the gear transmission. The role of the pulley is to wind or unwind the wire when the slider moves up or down. A spring is fixed on the body of the joint and connected to the winding pulley through an idle pulley mounted on the moving part of the prismatic joint. This element arrangement makes the moving part of the prismatic joint supported on two parallel segments of the wire and the tension in this wire is generated by the spring. This makes the spring force needed half the weight of the link.

$$2T = m\ g \tag{4}$$

where T is the tension force in the wire generated by the spring.

From kinematics of the system, the relationship between the joint's motion and the change in the wire's length as follows:

$$\Delta l = 2\ q \tag{5}$$

where q is the joint displacement and Δl is the corresponding change in the wire's length.

This means that the retraction or expansion of the wire's length should be twice the slider's displacement q. This dictates the transmission ratio between the pinion gear and the pulley to be 1:2.

III. Compensator for 2-DoF RP system

The goal here is to compensate gravity force for a 2-DoF RP system. Adding rotation increases the problem's complexity as the gravitational torques variate non-linearly with rotation and the prismatic joint presents a moving center of mass. A 2-Dof RP system is in Fig. 4. The system consists of a revolute joint and a prismatic joint. The system consists of two masses m_1 and m_2 at distances l_{c1} and l_{c2} from the center of rotation, respectively. The revolute joint rotates with angle q_1 and the prismatic joint slides with distance q_2. This makes the gravitational torque in the revolute joint as follows:

$$\tau_1 = (l_{c1}\ m_1 + l_{c2}\ m_2)\ g\ \cos(q_1) \tag{6}$$

where τ_1 is the torque effort of the revolute joint.

As the prismatic joint slides with value q_2, we can reformulate the variable l_{c2} as follows:

$$l_{c2} = l_{s_0} + q_2 \tag{7}$$

where l_{s_0} defines a minimum distance between m_2 and the center of rotation.

As for effort in the prismatic joint:

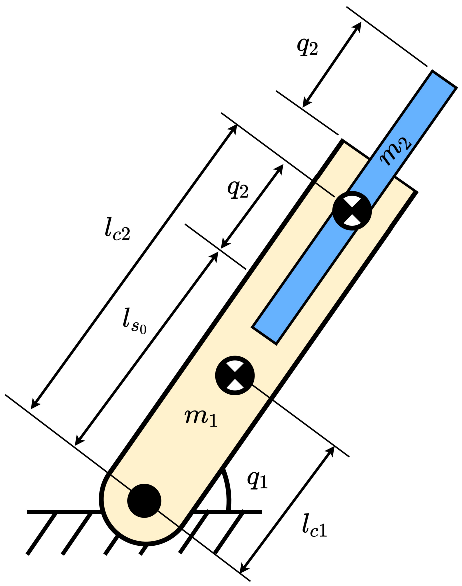


Fig. 4: A geometric representation of 2-DoF RP system

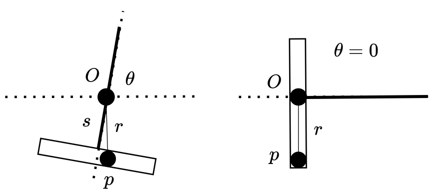


Fig. 5: A geometric representation of pin-slot mechanism

$$\tau_2 = m_2 \ g \ \sin(q_1) \tag{8}$$

where τ_2 is the force effort of the prismatic joint.

This equation shows that effort in the prismatic joint is non-linearly changing accordingly with the rotation angle of the first joint. The pin-slot mechanism shown in Fig. 5 is designed to compensate such non-linearity. The mechanism consists of a slot that rotates with angle θ around point O and moves linearly through point O. A pin p is fixed at a constant vertical distance r from point O and slides along the slot. As point p is fixed, the distance between the slot and point changes as follows:

$$s = r \ \sin(\theta) \tag{9}$$

where s is the distance between the slot and point O and r is the distance between points O and p.

We can use this mechanism to compensate gravity effort in the prismatic joint as shown in Fig. 6. The pin-slot mechanism is used to variate the tension in the spring according to rotation angle q_1. To statically balance prismatic joint effort, we can properly choose spring stiffness k and pin distance r.

$$k_1 \ r \ \sin(q_1) = \frac{1}{2} m_2 \ g \ \sin(q_1) \tag{10}$$

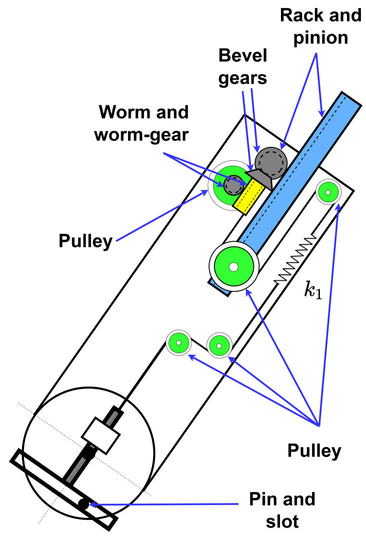


Fig. 6: A geometric representation of 2-DoF RP system with prismatic joint compensator

And accordingly, we can choose the value of spring stiffness k_1.

$$k_1 = \frac{m_2 \ g}{2 \ r} \tag{11}$$

Equations (10) and (11) show parameter selection to compensate gravity effort in prismatic joint at any orientation and the mechanism presented in Fig. 3b shows how to maintain constant spring force along prismatic joint's motion. Compensation for the revolute joint can be achieved using the construction shown in Fig. 7. We can compensate gravity torque in the revolute joint by connecting a spring between points A and B. Point A is fixed at a vertical distance a from the ground while point B has an initial displacement b from the center of rotation and sliding along the link. Point B is attached to a slider mounted on a pulley-belt mechanism. The belt moves the slider with distance $q*$ along the link which means sliding of point B is a ratio of the displacement of the prismatic joint which can be achieved by gear reduction. Another spring is connecting between points C and D where point C is fixed on vertical distance c and point D is fixed with distance d along the link . Which makes the vector representing the position of point B and D as follows:

$$\vec{B} = \left[(b+q*) \ \cos(q_1), \ (b+q*) \ \sin(q_1)\right]^T \tag{12}$$

$$\vec{D} = \left[d \ \cos(q_1), \ d \ \sin(q_1)\right]^T \tag{13}$$

To choose the proper value of the spring's stiffness, we need to satisfy equilibrium condition with sum of torques equals to zero. Torque generated by the springs can be calculating using the cross product $(\vec{B} \times \vec{BA})$ and $(\vec{D} \times \vec{DC})$.

$$\tau_{sp,2} = c \ d \ k_2 \ \cos(q_1) \tag{14}$$

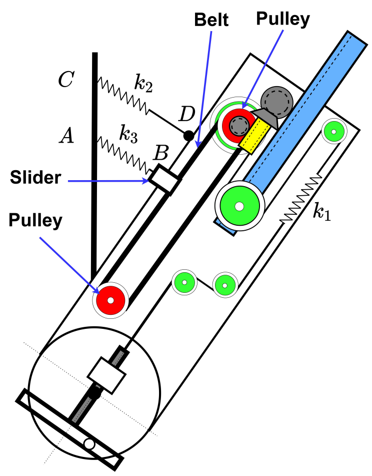


Fig. 7: A geometric representation of 2-DoF RP system with prismatic and revolute joints comepensators

where $\tau_{sp,1}$ is the torque generated by spring CD.

$$\tau_{sp,3} = a\ (b + q*)\ k_3\ \cos(q_1) \tag{15}$$

where $\tau_{sp,2}$ is the torque generated by spring AB.

From (6), (14) and (15), we can apply the equilibrium condition to calculate the value of springs' stiffness coefficients.

$$\tau_{sp,2} + \tau_{sp,3} = (l_{c1}\ m_1 + l_{c2}\ m_2)\ g\ \cos(q_1) \tag{16}$$

We can select our parameters to distribute the torque between both spring in a way that would eliminate the right-hand side of (16).

$$\tau_{sp,2} = l_{c1}\ m_1\ g\ \cos(q_1) \tag{17}$$

from this equation we can choose the value k_2.

$$k_2 = \frac{l_{c1}\ m_1\ g}{c\ d} \tag{18}$$

and similarly for the second spring while substituting with (7).

$$\tau_{sp,3} = (l_{s_0} + q_2)\ m_2\ g\ \cos(q_1) \tag{19}$$

which makes k_3 as follows:

$$k_3 = \frac{(l_{s_0} + q_2)}{(b + q*)} \frac{m_2\ g}{a} \tag{20}$$

For this equation to hold, the ratio between $(l_{s_0} + q_2)$ and $(b + q*)$ should be constant.

$$\frac{(l_{s_0} + q_2)}{(b + q*)} = \frac{m_2\ g}{a\ k_3} \tag{21}$$

This equation indicates that we can determine the distance b and the reduction ratio between q_2 and $q*$.

$$b = l_{s_0} \frac{a\ k_3}{m_2\ g}, \qquad q* = q_2 \frac{a\ k_3}{m_2\ g} \tag{22}$$

This means that we can control the span where point B can slide same as the location of this span. This gives freedom in realizing the mechanism which can be quite complex.

IV. Discussion

This system presents a concept for gravity compensation for prismatic joints for robotic systems. The concept depends on analytically decoupling joints' effort expressions and compensating them with equivalent mechanical mechanisms using linear springs. By coupling these mechanisms with the robotic system's joints, they can produce counter-balancing efforts that leads to static equilibrium without the need for actuator's effort.

Designing a gravity compensator for prismatic joints is challenging because of the moving center of mass. Unlike links coupled with revolute joints which has a defined center of mass, prismatic joints change the location of the center of mass which increases the non-linearity of the actuator's effort when moving along an inclined axis. The moving spring attachment point helps tackling this problem making the counter-balancing mechanism variate in proportion to motion span.

The use of worm gears is important for realizing this concept. When the worm gear is of self-locking type, it can hold reverse torque through friction. This ensures isolation of forces through the introduction of internal reaction force that blocks any backward torque. It acts as a one-way gate to torque as it can pass torque in one way and blocks reverse torque. Another advantage of worm gears is that they have high reduction ratio that reduces speed and magnifies the torque, which makes motion resistance less significant.

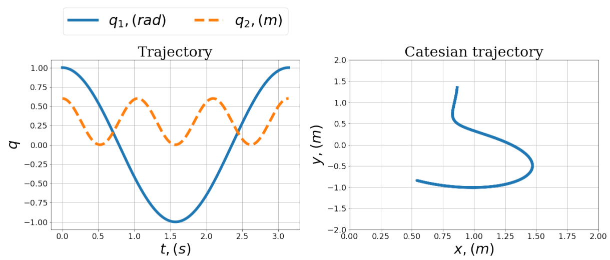
V. Results

A joint trajectory was tested in simulation. Parameters of manipulator and gravity compensator were assigned according to Table I. parameters a, b, c, and d were assigned arbitrarily while values k_1, k_2, k_3 were calculated from Eqs. (11), (18) and (20). Ratios b/l_{s_0} and $q* : q_2$ were calculated using Eq. (22). Simulation of joint torques show complete compensation of gravity force along the Cartesian trajectory.

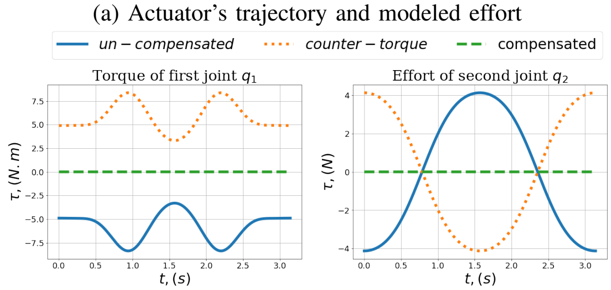
The value of spring coefficients depend on their mounting location on the manipulator. Moreover, mounting points of the springs can depend on the spring coefficient which gives more flexibility in the design process, especially, with limited space. However, the choice of the mounting points of the sliding spring is limited depending on the reduction ratio between the actuator's sliding range and the spring's mounting point sliding range. This means that the mounting location for the spring compensating the torque of the moving mass depends on the transmission ratio. It is practical to determine the mounting location prior to transmission ratio. Transmission ratio should reduce the span of motion to keep the sliding spring within the spacial limits of the link.

TABLE I: Design parameters used in simulation

parameter	value	unit
manipulator parameters		
l_1	mm	1000
l_2		500
l_{c1}		500
l_{s0}		250
l_{c2}		$l_{s0}+q_2$
m_1	kg	1
m_2		0.5
compensator parameters		
b/l_{s0}		1/2
$q^* : q_2$		1:2
a	mm	400
b		125
c		400
d		500
k_1	N/m	16.33
k_2		24.5
k_3		24.5



(a) Actuator's trajectory and modeled effort



(b) Counter effort and compensation

Fig. 8: Simulation results of effort and counter-balance

The relationship between the span of prismatic joint motion q_2 and the span of the spring connection point B is presented in Fig. 9. We can see how the transmission ratio can affect the sliding of point B. From the graph, we can see if the transmission ratio is $1:1$, point B needs to extend beyond the physical limits of the first link l_1. However, with higher transmission values, the motion span of point B gets smaller. The value of b determines the location of point B when $q_2=0$. As a design problem, either 20 or 22 can be used to determine the design parameters. A decision of either values k_3 or $(l_{s0}+q_2/(b+q*)$ can be made and then equations 20 or 22 can be used to determine the other parameter.

VI. CONCLUSION

This paper proposes a preliminary concept for a passively adapting gravity compensator dedicated for rotating prismatic joints. The compensator depends on tension springs and gear

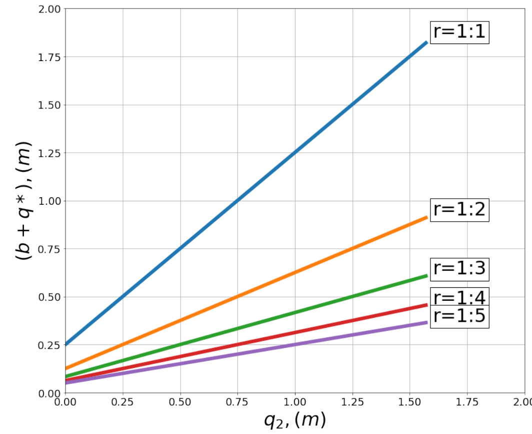


Fig. 9: Modeled relationship between prismatic joint distance q_2 and the span of point B based on different transmission ratios

transmission to produce corresponding counter-effort to compensate gravity. The design of the compensator's components is parameterized to suit different constructions of manipulators and space limitations. The sliding spring mounting point provides the ability to compensate gravity torque generated by a sliding mass at an angle. The wire retracting mechanism provides the ability to generate constant force along the sliding range of the linear actuator.

In the future, It is necessary to study dynamics of the system. Moreover, studying this concept with other common manipulator designs and analyzing stiffness properties of the resulting system.

ACKNOWLEDGMENT

This work was supported by Russian Scientific Foundation (Project number 22-41-02006).

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