1 Calculus

1)

3.1 Calculus

Problem 7. Find an integral:

$$\int \frac{1}{\sqrt{2\,x+1}}dx$$

(A):

$$1/\sqrt{2\,x+1}$$

2

(B):

$$\sqrt{2\,x}$$

(C):

$$1/\sqrt{2\,x}$$

(D):

$$\sqrt{2\,x+1}$$

2)

Problem 14. Find a derivative of the function:

$$y(x) = x^2 \log(x)\,\sin(x)\,.$$

(A):

$$y'(x) = x\sin(x) + 2\,x\log(x)\,\sin(x) + x^2\cos(x)\,\log(x)\,.$$

(B):

$$y'(x) = x\cos(x) + 2\,x\log(x)\,\sin(x) + x^2\cos(x)\,\log(x)\,.$$

(C):

$$y'(x) = x\sin(x) + x^2\cos(x)\,\log(x)\,.$$

(D):

$$y'(x) = x\sin(x) + 2\,x\log(x)\,\cos(x) + x^2\sin(x)\,\log(x)\,.$$

2 Exponents and complex values

3)

Problem 13. Write in the trigonometric form the following complex number:

$$z = 1$$

(A):

$$z = 1\,(\mathrm{i}\sin(0))\,.$$

(B):

$$z = 1\,(\cos(0) + \mathrm{i}\sin(0))\,.$$

(C):

$$z = 1\,(\cos(0))\,.$$

(D):

$$z = 1.$$

4)

Problem 16. Find the real and imaginary part of the following complex number:

$$z = 2 + \frac{2+5\,\mathrm{i}}{3+5\,\mathrm{i}}$$

(A):

$$z = \frac{101}{34} + \frac{5}{34}\,\mathrm{i}.$$

(B):

$$z = \frac{99}{34} + \frac{5}{34}\,\mathrm{i}.$$

(C):

$$z = \frac{99}{35} + \frac{5}{36}\,\mathrm{i}.$$

(D):

$$z = \frac{99}{33} + \frac{5}{36}\,\mathrm{i}.$$

3 Series

5)

Problem 1. Find the Taylor series for the function

$$y(x) = \frac{1}{(x+1)^5}.$$

around $x = 0$.

(A):

$$y(x) \approx \ldots - 126\,x^5 + 70\,x^4 - 35\,x^3 + 15\,x^2 - 9\,x + 1.$$

(B):

$$y(x) \approx \ldots - 126\,x^5 + 50\,x^4 - 35\,x^3 + 15\,x^2 - 5\,x + 1.$$

(C):

$$y(x) \approx \ldots - 126\,x^5 + 70\,x^4 - 25\,x^3 + 15\,x^2 - 5\,x + 1.$$

(D):

$$y(x) \approx \ldots - 126\,x^5 + 70\,x^4 - 35\,x^3 + 15\,x^2 - 5\,x + 1.$$

6)

Problem 5. Find the Maclaurin series for the function

$$y(x) = \log\left(\frac{1+x}{1-x}\right).$$

(A):

$$y(x) \approx \ldots \frac{2\,x^8}{9} + \frac{2\,x^4}{5} + \frac{2\,x^2}{3} + 3\,x.$$

(B):

$$y(x) \approx \ldots \frac{2\,x^7}{9} + \frac{2\,x^5}{7} + \frac{2\,x^3}{3} + x.$$

(C):

$$y(x) \approx \ldots \frac{2\,x^7}{7} + \frac{2\,x^5}{5} + \frac{2\,x^3}{3} + 2\,x.$$

(D):

$$y(x) \approx \ldots \frac{2\,x^7}{9} + \frac{2\,x^5}{7} + \frac{2\,x^3}{5} + 2\,x.$$

4 Linear Algebra

7)

Problem 1. Express all solutions of the following system of equations:

$$\begin{cases} 2x_1 - 3x_2 - 5x_3 = 0 \\ 2x_1 + x_2 = 0 \end{cases}$$

- (A) $x_1 = 0,\ x_2 = 0,\ x_3 = 0$
- (B) $x_1 = 5t,\ x_2 = -10t,\ x_3 = 8t,\ \forall t \in \mathbb{R}$
- (C) $x_1 = 2,\ x_2 = 1,\ x_3 = 8/5$
- (D) $x_1 = 10,\ x_2 = -20,\ x_3 = 16$

8)

Problem 2.

Find projection of a vector $v = \begin{bmatrix} 5 & 10 & -\sqrt{5} \end{bmatrix}$ onto the plane span by vectors $e_1 = \begin{bmatrix} 3 & 4 & 0 \end{bmatrix}$ and $e_2 = \begin{bmatrix} -4 & 2 & \sqrt{5} \end{bmatrix}$ and express it in the original basis.

- (A) $p = \begin{bmatrix} 5 & 10 & 5 \end{bmatrix}$
- (B) $p = \begin{bmatrix} 5 & 10 & 0 \end{bmatrix}$
- (C) $p = \begin{bmatrix} 37 & 42 & -\sqrt{5} \end{bmatrix}$
- (D) $p = \begin{bmatrix} 29 & 46 & \sqrt{5} \end{bmatrix}$

9)

Problem 9.

Calculate:

$$\begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

(A):

$$\begin{pmatrix} 3 & -1 \\ 5 & -1 \end{pmatrix}$$

(B):

$$\begin{pmatrix} 3 & -1 \\ 1 & -1 \end{pmatrix}$$

(C):

$$\begin{pmatrix} 1 & -1 \\ 5 & -1 \end{pmatrix}$$

(D):

$$\begin{pmatrix} 3 & -3 \\ 5 & -1 \end{pmatrix}$$

10)

Problem 18.

Calculate the determinant:

$$\begin{vmatrix} a & a & a \\ -a & a & x \\ -a & -a & x \end{vmatrix}.$$

(A):

$$2\,a^3 + 2\,x\,a^2$$

(B):

$$2\,a^3 + x\,a^2$$

(C):

$$a^3 + 2\,x\,a^2$$

(D):

$$3\,a^3 + 3\,x\,a^2$$

5 Analytic geometry

11)

Problem 1.

Three points A, B, C are given on the plane. Find the area of the triangle formed by these points. $A(-1,-1)$, $B(2,5)$, $C(-2,7)$

12)

Problem 7.

Find the value of the parameter m at which the points $(9,5,8)$ and $(-3,7,2)$ are symmetric about the plane $6x - y + 3z = m$

6 Differential equations

13)

Problem 1. Find solution the following equation as a function of time t and initial conditions x_0:

$$\dot{x} = 5 - x$$

- (A) $x = 5e^t$
- (B) $x = Ce^{-t} + x_0$
- (C) $x = Ce^{-t-5} + x_0$
- (D) $x = (x_0 - 5)e^{-t}$

14)

Problem 2. Does this equation converges or diverges?

$$\ddot{x} + 7\dot{x} + 3x = 0$$

A. Converges to $\frac{7}{3}$
B. Converges to 0
C. Diverges to $-\infty$
D. Diverges to ∞

15)

Problem 4. Show that the function $y = Cx^2$, where C is a constant, is the solution of the following DE. Find a particular solution with following initial condition $y_0(1) = 3$:

$$x \frac{\partial}{\partial x} y(x) - 2\,y(x) = 0$$

(A):

$$y_0 = 3\,x^3.$$

(B):

$$y_0 = 3\,x^4.$$

(C):

$$y_0 = 2\,x^2.$$

(D):

$$y_0 = 3\,x^2.$$

16)

Solve the DE

$$x \frac{\partial}{\partial x} y(x) = \frac{y(x)}{\log(x)},$$

under the initial condition $y(e) = 1$.

(A):

$$y = e \log(x)$$

(B):

$$y = 3\,\frac{1}{x}$$

(C):

$$y = 2 \log(x)$$

(D):

$$y = \log(x)$$